LTI ODE Continued

Lecture #4

BME 333 Biomedical Signals and Systems
- J.Schesser

Homework

- 2^{st} Order ODEs

1. Using Matlab, plot the response for following systems. Identity what type of system each is. Submit your code:

$$a)\ddot{x}+10\dot{x}+4x=0; x(0)=5; \dot{x}(0)=0$$

$$b)\ddot{x}+4\dot{x}+4x=0; x(0)=5; \dot{x}(0)=0$$

$$c)\ddot{x}+1\dot{x}+4x=0; x(0)=5; \dot{x}(0)=0$$

$$d)\ddot{x}+4x=0; x(0)=5; \dot{x}(0)=0$$

2. 2.2 An LTI system is described by the second-order ODE:

$$\ddot{y}+7\dot{y}+10y=x(t)$$

   a. Use the p operator notation to find the roots of the characteristic equation.
   b. Assume y(0)=0, dy(0)/dt=9 and x(t)=0, find y(t).
   c. Now Let x(t) = 10 and same initial conditions, find y(t).

Homework

3. A quadratic low-pass filter is described by the second-order ODE:

$$\ddot{y} + (2\xi\omega_n)\dot{y} + \omega_n^{\,2} y = x(t)$$

a. The characteristic equation for this ODE in terms of the p operator has complex conjugate roots. Find an algebraic expression for the position of the roots.

b. Let $x(t)=1$. Find the steady-state output.

c. Let $x(t)=0$, $y(0)=0$ and $dy(0)/dt=10$, Find and sketch $y(t)$ for $\xi = .5$ and $\omega_n=1$

Homework

4. An alternate way of writing the ODE for an underdamped system is

$$\ddot{y} + 2a\dot{y} + (b^2 + a^2)y = x(t)$$

Let a=.5 and b=.86603. Repeat a, b, and c in problem 2.

Homework

5.BioSignals

- An heart signal is sampled at the rate 250 s/s and is passed to EKG which has an input consisting of a low pass filter. The filter is a resistor and capacitor in series where the output of the filter is taken across the capacitor. What should be the value of the Capacitor if the Resistor is 1k ohms and the time constant of the filter so that the transient response is completed within 1/10 of the sample time? What is the cutoff frequency of this filter?

Homework

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body:

$$\ddot{y} + \omega_n^2 y = x(t)$$

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.

c. Use Matlab to graph the signals for both parts. Assume $\omega_n=2\pi$.

Homework Answers #1

- 2nd Order ODEs

$$a)\ \ddot{x}(t)+10\dot{x}(t)+4x(t)=0$$

$$[p^2+10p+4]x(t)=0$$

$$p_{1,2}=\frac{-10\pm\sqrt{100-16}}{2}=\frac{-10\pm\sqrt{84}}{2}=\frac{-10\pm 9.2}{2}=-9.6,-0.42$$

$$x(t)=K_1e^{-9.6t}+K_2e^{-0.42t}$$

$$x(0)=5=K_1+K_2;$$

$$\dot{x}(0)=0=-9.6K_1-0.42K_2\Rightarrow K_2=-\frac{9.6}{0.42}K_1=-22.9K_1$$

$$K_1+K_2=K_1-22.9K_1=-21.9K_1=5\Rightarrow K_1=-\frac{5}{21.9};K_1=-0.23$$

$$K_2=5+0.23=5.23$$

$$x(t)=-0.23e^{-9.6t}+5.23e^{-0.42t}$$

Homework Answers #1

- 2nd Order ODEs

$$b)\ \ddot{x}(t) + 4\dot{x}(t) + 4x(t) = 0$$

$$[p^2 + 4p + 4]x(t) = 0$$

$$p_{1,2} = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4}{2} = -2$$

$$x(t) = K_1 t e^{-2t} + K_2 e^{-2t}$$

$$x(0) = 5 = K_2;$$

$$\dot{x}(t) = -2K_1 t e^{-2t} + K_1 e^{-2t} - 2K_2 e^{-2t}$$

$$\dot{x}(0) = -0 + K_1 - 2K_2 = K_1 - 10 = 0 \Rightarrow K_1 = 10$$

$$x(t) = 10te^{-2t} + 5e^{-2t}$$

Homework Answers #1

- 2^{nd} Order ODEs

$$c)\ \ddot{x}(t)+1\dot{x}(t)+4x(t)=0$$

$$[p^2+1p+4]x(t)=0$$

$$p_{1,2}=\frac{-1\pm\sqrt{1-16}}{2}=\frac{-1\pm\sqrt{-15}}{2}=\frac{-1\pm j\sqrt{15}}{2}=\frac{-1\pm j3.87}{2}=-.5\pm j1.93$$

$$p_1=-0.5+j1.93=1.99e^{j1.8};\ p_2=-0.5-j1.93=p_1{*}=1.99e^{-j1.8}$$

$$x(t)=K_1e^{(-.5+j1.93)t}+K_2e^{(-.5-j1.93)t}$$

$$K_1=a+jb=re^{j\theta}$$

$$K_2=K_1{*}=a-jb=re^{-j\theta}$$

$$x(t)=K_1e^{(-0.5+j1.93)t}+K_1{*}e^{(-0.5-j1.93)t}=e^{-0.5t}(re^{j\theta}e^{j1.93t}+re^{-j\theta}e^{-j1.93t})$$

$$=2re^{-0.5t}\cos(1.93t+\theta)$$

$$x(0)=5=K_1+K_2;$$

$$5=K_1+K_2=K_1+K_1{*}=2\Re e\{K_1\}=2a\Rightarrow a=2.5$$

$$\dot{x}(0)=sK_1+s^*K_2=sK_1+s^*K_1{*}=1.99e^{j1.8}re^{j\theta}+1.99e^{-j1.8}re^{-j\theta}=0$$

$$e^{j(1.8+\theta)}=-e^{-j(1.8+\theta)}=e^{\pm j\pi}e^{-j(1.8+\theta)}=e^{-j(1.8+\theta\pm\pi)}$$

$$-1.8+\theta=-1.8-\theta\pm\pi$$

$$2\theta=-3.6\pm\pi\Rightarrow\theta=-1.8\pm\pi/2$$

$$\text{if }\theta=-1.8+\pi/2=-0.23;\ r\cos(-1.8+\pi/2)=r(0.97)=2.5\Rightarrow r=2.57\Rightarrow K_1=2.57e^{-j0.23}$$

$$\text{if }\theta=-1.8-\pi/2=-3.4;\ r\cos(-1.8-\pi/2)=r(-0.97)=2.5\Rightarrow r=-2.57\Rightarrow K_1=-2.57e^{-j3.4}=2.57e^{-j0.23}$$

$$x(t)=2(2.57)e^{-0.5t}\cos(1.93t-0.23)=5.14e^{-0.5t}\cos(1.93t-0.23)$$

CHECK

$$x(0)=5.14e^{-0.5x0}\cos(1.93(0)-0.23)=5.14\cos(-0.23)=5;$$

$$\dot{x}(t)=(-0.5)\times5.14e^{-0.5t}\cos(1.93t-0.23)+5.14(-1.93)e^{-0.5t}\sin(1.93t-0.23)$$

$$\dot{x}(0)=5.16(-.5)e^{-.5\times0}\cos[1.93\times0-0.23]-5.16(1.93)e^{-.5\times0}\sin[1.93\times0-0.23]$$

$$=5.16(-.5)\cos[-.23]-5.16(1.93)\sin[-.23]=-2.5-(-2.5)=0$$

Homework Answers #1

- 2^{nd} Order ODEs

$$d)\ \ddot{x}(t) + 4x(t) = 0$$

$$[p^2 + 4]x(t) = 0$$

$$p_{1,2} = \frac{\pm\sqrt{-16}}{2} = \pm j2$$

$$x(t) = K_1 e^{j2t} + K_2 e^{-j2t}$$

$$x(0) = 5 = K_1 + K_2;$$

$$\dot{x}(0) = j2K_1 - j2K_2 = 0 \Rightarrow K_2 = K_1$$

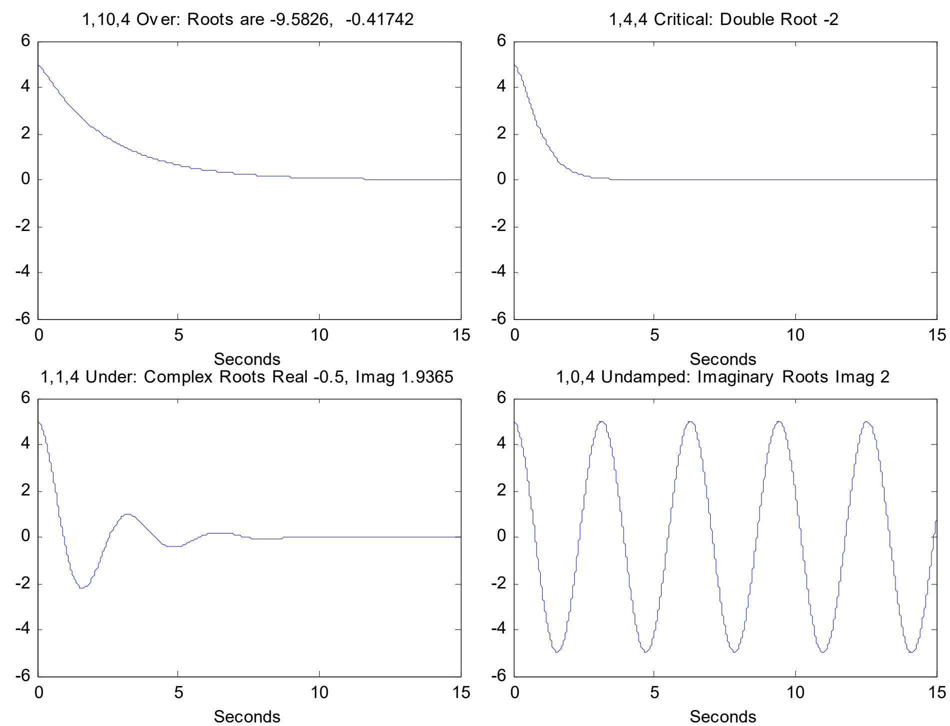
$$K_1 + K_2 = 2K_1 = 5; K_1 = K_2 = 2.5$$

$$x(t) = 2.5e^{j2t} + 2.5e^{-j2t} = 5\cos[2t]$$

$$x(0) = 5\cos[2 \times 0] = 5; \dot{x}(0) = -10\sin[2 \times 0] = 0$$

Matlab Code

```
clear all;
coefficent=[1 10 4];
[r1,r2,type,x,time]=second(coefficent,5,0);
plotsecond(time,x,[.1 .55 .4 .35],coefficent,type,r1,r2);
coefficent=[1 4 4];
[r1,r2,type,x,time]=second(coefficent,5,0);
plotsecond(time,x,[.55 .55 .4 .35],coefficent,type,r1,r2);
coefficent=[1 1 4];
[r1,r2,type,x,time]=second(coefficent,5,0);
plotsecond(time,x,[.1 .1 .4 .35],coefficent,type,r1,r2);
coefficent=[1 0 4];
[r1,r2,type,x,time]=second(coefficent,5,0);
plotsecond(time,x,[.55 .1 .4 .35],coefficent,type,r1,r2);
```



Matlab Code

```
function [r1,r2,type,x,time]=second(coefficent,ic,fdic);
discriminant=coefficent(2)^2-4*coefficent(1)*coefficent(3);
time=(0:.01:15);
if discriminant>0
   [r1,r2]=rootsreal(coefficent,discriminant);
  A1=(r2*ic-fdic)/(r2-r1);A2=+(fdic-r1*ic)/(r2-r1);
   x1=A1*exp(r1*time);x2=A2*exp(r2*time);
   x=x1+x2;
   type=1;
   else
   if discriminant==0
      r1=rootsdouble(coefficent);r2=0;
     A=ic;B=fdic-r1*ic;
      for i=1:length(time);
         x(i)=(A+B*time(i))*exp(r1*time(i));
      end
      type=2;
   else
      [r1,r2]=rootscomplex(coefficent,discriminant);
      real=r1;
      imag=r2;
      tc=-real;
      if tc==0
         tc=imag/(2*pi)*3;
      end
      B=atan(-(fdic-ic*real)/(imag*ic));
      A=ic/cos(B);
      for i=1:length(time);
         x(i)=A*exp(real*time(i))*cos(imag*time(i)+B);
      end
      type=3;
   end
end
```

Matlab Code

```
function [r1,r2]=rootsreal(a,d)
r1=-a(2)-sqrt(d);r1=r1/2;
r2=-a(2)+sqrt(d);r2=r2/2;

function r=rootsdouble(a)
r=-a(2)/2;

function [real,imag]=rootscomplex(a,d)
real=-a(2)/2;
imag=sqrt(-d)/2;
```

Matlab Code

```
function plotsecond(time,x,positionpp,coefficent,type,r1,r2)
subplot('position',positionpp);
plot(time,x);
set(gca,'FontSize',7);
xlabel('Seconds');
axis([0 15 -6 6]);
tt=titlecal(coefficent,type,r1,r2);
title(tt);
```

Matlab Code

```
function tt=titlecal(coefficent,type,r1,r2);
if type==1
tt=([num2str(coefficent(1)),',',num2str(coefficent(2)),',',num2str(coefficent(3)),' Over: Roots are
      ',num2str(r1),',  ',num2str(r2)]);
else
   if type==2
      tt=([num2str(coefficent(1)),',',num2str(coefficent(2)),',',num2str(coefficent(3)),' Critical: Double Root
      ',num2str(r1)]);
    else
      if type==3
         if r1==0
         tt=([num2str(coefficent(1)),',',num2str(coefficent(2)),',',num2str(coefficent(3)),' Undamped:
      Imaginary Roots Imag ',num2str(r2)]);
      else
         tt=([num2str(coefficent(1)),',',num2str(coefficent(2)),',',num2str(coefficent(3)),' Under: Complex
      Roots Real ',num2str(r1),', Imag ',num2str(r2)]);
         end
      else
         tt=(['ERROR']);
      end
   end
end
```

Homework Answers #1

- 2nd Order ODEs
  - Prob 2 plus for 2.2b assume *x(t)=0* with *y(0)=0, dy(0)/dt=9* and for 2.2c assume *x(t)=10*

$$a)\ \ddot{y}(t)+7\dot{y}(t)+10y(t)=x(t)$$
$$[p^2+7p+10]y(t)=x(t)$$
$$p_{1,2}=\frac{-7\pm\sqrt{49-40}}{2}=\frac{-7\pm 3}{2}=-5,-2$$

$$b)\ x(t)=0$$
$$y(t)=K_1e^{-5t}+K_2e^{-2t}$$
$$y(0)=0;$$
$$K_1+K_2=0;\ K_1=-K_2$$
$$\dot{y}(0)=9;$$
$$-5K_1+-2K_2=9;-5K_1+2K_1=9;K_1=-3$$
$$y(t)=3(e^{-2t}-e^{-5t})$$

$$c)\ x(t)=10$$
$$y(t)=K_1e^{-5t}+K_2e^{-2t}+K_3$$
$$y(0)=0;\ \therefore K_1+K_2+K_3=0$$

Substituting the component of the response due to the source into the differential eqn.

$$0+7\times 0+10y(t)=10;\ \therefore K_3=1,\ \&\ K_1+K_2=-1;$$
$$\therefore K_2=-(1+K_1)$$
$$\dot{y}(0)=9;\ \therefore -5K_1-2K_2=9;\therefore -5K_1-2\times-(1+K_1)=9$$
$$-5K_1+2+2K_1=9;$$
$$K_1=-\frac{7}{3};K_2=\frac{4}{3}$$
$$y(t)=1-(\frac{7}{3}e^{-5t}-\frac{4}{3}e^{-2t})$$

Homework Answers #2

- 2nd Order ODEs
  - Prob. 3a

$$\ddot{y}(t) + (2\zeta\omega_n)\dot{y}(t) + \omega_n{}^2 y(t) = x(t)$$

Since roots are complex conjugates, then

$(2\zeta\omega_n)^2 < 4\omega_n{}^2$ and

$$p_{1,2} = \frac{-(2\zeta\omega_n) \pm j\sqrt{4\omega_n{}^2 - (2\zeta\omega_n)^2}}{2}$$

$$= -(\zeta\omega_n) \pm j\sqrt{\omega_n{}^2 - (\zeta\omega_n)^2}$$

$$= -\zeta\omega_n \pm j\beta\omega_n; \beta = \sqrt{1-(\zeta)^2}$$

Homework Answers #3

- 2[nd] Order ODEs
  - Prob. 3b assume $x(t)=1$

$\ddot{y}+(2\xi\omega_n)\dot{y}+\omega_n{}^2 y=1$

$p_{1,2}=-\zeta\omega_n \pm j\beta\omega_n$

$y(t)=e^{-(\zeta\omega_n)t}(K_1 e^{+j\beta\omega_n t}+K_2 e^{-j\beta\omega_n t})+K_3;$

The steady state output is the solution associated with the response due to the source.

$\therefore 0+(2\xi\omega_n)0+\omega_n{}^2 y=x(t)=1$

$y(t)=\frac{1}{\omega_n{}^2}=K_3$

Although not asked for let's calculate the rest of the solution and assume $y(0)=0$ and $\frac{dy(0)}{dt}=10$, then

$y(0)=K_1+K_2+K_3=K_1+K_2+\frac{1}{\omega_n{}^2}=0; K_2=-(\frac{1}{\omega_n{}^2}+K_1)$

$\dot{y}(t)=(-\zeta\omega_n+j\beta\omega_n)K_1 e^{(-\zeta\omega_n+j\beta\omega_n)t}+(-\zeta\omega_n-j\beta\omega_n)K_2 e^{(-\zeta\omega_n-j\beta\omega_n)t}+0$

$\dot{y}(0)=(-\zeta\omega_n+j\beta\omega_n)K_1+(-\zeta\omega_n-j\beta\omega_n)K_2=p_1K_1+p_1{}^*K_2=10$

$p_1K_1+p_1{}^*\times-(\frac{1}{\omega_n{}^2}+K_1)=10; p_1K_1-p_1{}^*K_1=10+\frac{p_1{}^*}{\omega_n{}^2}$

$$K_1=\frac{10+\frac{p_1{}^*}{\omega_n{}^2}}{p_1-p_1{}^*}; K_2=-(\frac{1}{\omega_n{}^2}+\frac{10+\frac{p_1{}^*}{\omega_n{}^2}}{p_1-p_1{}^*})=-\frac{\frac{p_1-p_1{}^*}{\omega_n{}^2}+10+\frac{p_1{}^*}{\omega_n{}^2}}{p_1-p_1{}^*}=-\frac{10+\frac{p_1}{\omega_n{}^2}}{p_1-p_1{}^*}$$

Since $p_1-p_1{}^*=-\zeta\omega_n+j\beta\omega_n-(-\zeta\omega_n-j\beta\omega_n)=2j\beta\omega_n$

$$K_1=\frac{10+\frac{-\zeta\omega_n-j\beta\omega_n}{\omega_n{}^2}}{2j\beta\omega_n}=\frac{10\omega_n{}^2-\zeta\omega_n-j\beta\omega_n}{2j\beta\omega_n{}^3}=-\frac{1}{2\omega_n{}^2}-j\frac{(10\omega_n{}^2-\zeta\omega_n)}{2\beta\omega_n{}^3}$$

$$=\frac{1}{2\omega_n{}^2}(-1-j\frac{10\omega_n-\zeta}{\beta})=\frac{1}{2\omega_n{}^2}\sqrt{1+(\frac{10\omega_n-\zeta}{\beta})^2}\angle\tan^{-1}(\frac{10\omega_n-\zeta}{\beta})$$

$$K_2=-\frac{10+\frac{-\zeta\omega_n+j\beta\omega_n}{\omega_n{}^2}}{2j\beta\omega_n}=-\frac{10\omega_n{}^2-\zeta\omega_n+j\beta\omega_n}{2j\beta\omega_n{}^3}=-\frac{1}{2\omega_n{}^2}+j\frac{(10\omega_n{}^2-\zeta\omega_n)}{2\beta\omega_n{}^3}=K_1{}^*$$

$$y(t)=e^{-(\zeta\omega_n)t}(K_1e^{+j\beta\omega_n t}+K_1{}^*e^{-j\beta\omega_n t})+\frac{1}{\omega_n{}^2}$$

$$=e^{-(\zeta\omega_n)t}\frac{1}{2\omega_n{}^2}\sqrt{1+(\frac{10\omega_n-\zeta}{\beta})^2}\cos[\beta\omega_n t+\tan^{-1}(\frac{10\omega_n-\zeta}{\beta})]+\frac{1}{\omega_n{}^2}$$

Homework Answers #4

- 2nd Order ODEs
  - Prob. 3c assume $x(t)=0$ with y(0)=0 and dy(0)/dt=10

$$p_{1,2} = -\zeta\omega_n \pm j\beta\omega_n$$

$$y(t) = e^{-(\zeta\omega_n)t}(K_1 e^{+j\beta\omega_n t} + K_2 e^{-j\beta\omega_n t});$$

$$y(0) = 0; \quad \therefore K_1 + K_2 = 0; K_1 = -K_2$$

$$\dot{y}(0) = 10; \quad \therefore [-\zeta\omega_n + j\beta\omega_n]K_1 + [-\zeta\omega_n - j\beta\omega_n]K_2$$

$$= \mathbf{a}K_1 - \mathbf{a}^* K_1 = 10$$

$$K_1 = \frac{10}{\mathbf{a} - \mathbf{a}^*} = \frac{10}{2j\beta\omega_n}$$

$$K_2 = -\frac{10}{2j\beta\omega_n}$$

$$y(t) = e^{-(\zeta\omega_n)t}[\frac{10}{2j\beta\omega_n}(e^{+j\beta\omega_n t} - e^{-j\beta\omega_n t})]$$

$$= e^{-(\zeta\omega_n)t}\frac{10}{\beta\omega_n}\sin\beta\omega_n t$$

$$= e^{-.5t}\frac{10}{.866025}\sin .866025\, t$$

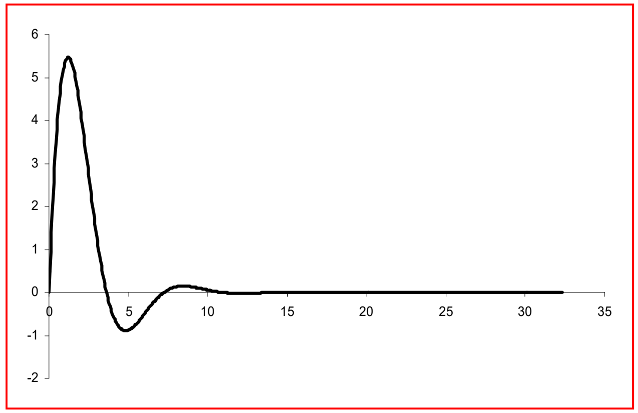
$$p_{1,2} = -\zeta\omega_n \pm j\beta\omega_n$$

$$\zeta = .5$$

$$\beta = \sqrt{1-.5^2} = \sqrt{.75} = .866025$$

$$\omega_n = 1$$

$$p_{1,2} = -.5 \pm j.866025$$



Homework Answers #5

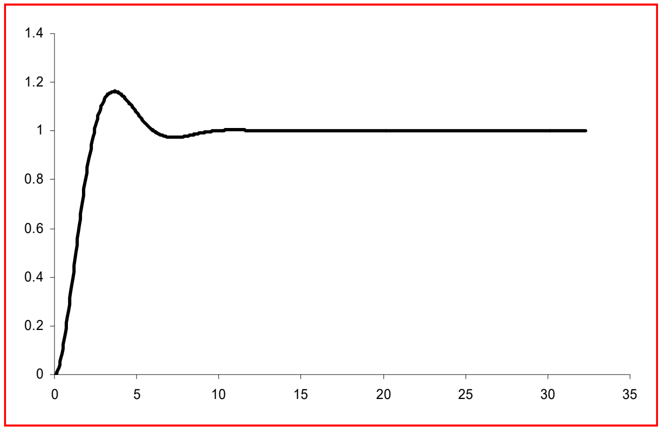
- 2nd Order ODEs
  - Prob. 4a

$$\ddot{y} + 2a\dot{y} + (b^2 + a^2)y = x$$

$$p_{1,2} = \frac{-2a \pm j\sqrt{4(b^2 + a^2) - 4a^2}}{2} = -a \pm jb$$

$$= -.5 \pm j.86603$$

  - Prob 4b



$$y(t) = 1 - \frac{e^{-(\zeta\omega_n)t}}{\omega_n^{\ 2}}[(\cos\beta\omega_n t + (\zeta/\beta)\sin\beta\omega_n t)]$$

$$= 1 - e^{-.5t}[(\cos .86603t + .577\sin .86603t)]$$

  - Prob 4c see Prob 3c

Homework

- BioSignals
  - An heart signal is sampled at the rate 250 s/s and is passed to EKG which has an input consisting of a low pass filter. The filter is a resistor and capacitor in series where the output of the filter is taken across the capacitor. What should be the value of the Capacitor if the Resistor is 1k ohms and the time constant of the filter so that the transient response is completed within 1/10 of the sample time? What is the cutoff frequency of this filter?

If the sample rate is 250 s/s, then the sampling period is 1/250 = 0.004 = 4 msec. One tenth of that time is 0.4 msec or 400 microsec. For this filter, the output yields a first order differential equation and if we want the transient response to be over within 400 microsec, then the time constant is 400/3 = 133 microsec. The time constant is RC and if R = 1k then C= 0.133 microfarads. The cutoff frequency of the filter is 1194 Hz.

$$V_{in}(t) = Ri(t) + \frac{1}{C}\int i(t)dt \Rightarrow V_{in}(t) = Ri(t) + \frac{1}{pC}i(t)$$

Homogeneous Equation

$$Ri(t) + \frac{1}{pC}i(t) = 0$$

Characteristic Equation

$$p + \frac{1}{RC} = 0$$

Source Free Response

$$i(t) = Ae^{-\frac{t}{RC}}$$

Homework Answers #1

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body: $\ddot{y} + \omega_n^2 y = x(t)$

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.

$$a)\ \ddot{y}(t) + \omega_n^2 y(t) = x(t)$$
$$[p^2 + \omega_n^2]y(t) = x(t)$$
$$p_{1,2} = \pm j\omega_n$$

$$x(t) = 0$$
$$y(t) = K_1 e^{j\omega_n t} + K_2 e^{-j\omega_n t}$$
$$y(0) = 0;$$
$$K_1 + K_2 = 0;\ K_1 = -K_2$$
$$y(t) = K_1 e^{j\omega_n t} - K_2 e^{-j\omega_n t}$$
$$\dot{y}(0) = \omega_n;$$
$$j\omega_n K_1 - j\omega_n K_2 = j\omega_n K_1 + j\omega_n K_1 = 2j\omega_n K_1 = \omega_n$$
$$K_1 = \frac{1}{2j} = 0.5\angle -\frac{\pi}{2}$$
$$y(t) = 0.5(e^{j(\omega_n t - \frac{\pi}{2})} + e^{-j(\omega_n t - \frac{\pi}{2})}) = 2\Re e\{0.5e^{j(\omega_n t - \frac{\pi}{2})}\}$$
$$= \cos(\omega_n t - \frac{\pi}{2}) = \sin(\omega_n t)$$

Homework Answers #1

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body:

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.

$$b)\ \ddot{y}(t)+\omega_n^2 y(t)=\cos\omega_n t$$

$$y(t)=K\cos(\omega_n t+\theta)$$

$$\ddot{y}(t)+\omega_n^2 y(t)=\Re e\{e^{j\omega_n t}\}$$

$$y(t)=\Re e\{Ke^{j(\omega_n t+\theta)}\}$$

$$\frac{d^2}{dt^2}\Re e\{Ke^{j(\omega_n t+\theta)}\}+\omega_n^2\Re e\{Ke^{j(\omega_n t+\theta)}\}$$

$$=\Re e\{\frac{d^2}{dt^2}Ke^{j(\omega_n t+\theta)}+\omega_n^2 Ke^{j(\omega_n t+\theta)}\}=\Re e\{e^{j\omega_n t})$$

$$\frac{d^2}{dt^2}Ke^{j(\omega_n t+\theta)}+\omega_n^2 Ke^{j(\omega_n t+\theta)}=e^{j\omega_n t}$$

$$-\omega_n^2 Ke^{j(\omega_n t+\theta)}+\omega_n^2 Ke^{j(\omega_n t+\theta)}=0\neq e^{j\omega_n t}$$

Homework Answers #1

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body:

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.

$$b)\ y(t) = (K_1 + K_2 t)\cos(\omega_n t + \theta) = (K_1 + K_2 t)\Re e\{e^{j(\omega_n t+\theta)}\}$$

$$\dot{y}(t) = K_2 \Re e\{e^{j(\omega_n t+\theta)}\} + (K_1 + K_2 t)\Re e\{j\omega_n e^{j(\omega_n t+\theta)}\}$$

$$\ddot{y}(t) = K_2 \Re e\{j\omega_n e^{j(\omega_n t+\theta)}\} + K_2 \Re e\{j\omega_n e^{j(\omega_n t+\theta)}\} + (K_1 + K_2 t)\Re e\{-\omega_n^2 e^{j(\omega_n t+\theta)}\}$$

$$= 2K_2 \Re e\{j\omega_n e^{j(\omega_n t+\theta)}\} + (K_1 + K_2 t)\Re e\{-\omega_n^2 e^{j(\omega_n t+\theta)}\}$$

$$2K_2 \Re e\{j\omega_n e^{j(\omega_n t+\theta)}\} + (K_1 + K_2 t)\Re e\{-\omega_n^2 e^{j(\omega_n t+\theta)}\} + \omega_n^2 (K_1 + K_2 t)\Re e\{e^{j(\omega_n t+\theta)}\}$$

$$= 2K_2 \Re e\{j\omega_n e^{j(\omega_n t+\theta)}\} = \Re e\{e^{j\omega_n t})$$

Homework Answers #1

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body:

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.

$$b)\ 2K_2 j\omega_n e^{j\theta} = 1$$

$$K_2 e^{j\theta} = \frac{1}{2j} = \frac{1}{2}e^{-j\frac{\pi}{2}};$$

$$K_2 = \frac{1}{2}; \theta = -\frac{\pi}{2};$$

$$y(t) = (K_1 + 0.5t)\cos(\omega_n t - \frac{\pi}{2}) = (K_1 + 0.5t)\sin(\omega_n t)$$

$$y(0) = 0 = (K_1 + 0.5\times 0)\times 0$$

$$\dot{y}(0) = 0.5\Re e\{e^{j(\omega_n 0 - \frac{\pi}{2})}\} + (K_1 + 0.5\times 0)\Re e\{j\omega_n e^{j(\omega_n 0 - \frac{\pi}{2})}\}$$

$$= 0.5\Re e\{e^{-j\frac{\pi}{2}}\} + K_1\Re e\{j\omega_n e^{-j\frac{\pi}{2}}\} = 0 + K_1\omega_n = \omega_n$$

$$K_1 = 1$$

$$y(t) = (1 + 0.5t)\cos(\omega_n t - \frac{\pi}{2}) = (1 + 0.5t)\sin(\omega_n t)$$

Homework Answers #1

6. Respiration may be modeled with the following second order equation, where $y(t)$ is the respiration signal and $x(t)$ is the additional load above the resting respiration on the body:

a. Calculate the resting respiration rate. Assume $y(0)=0$ and $dy(0)/dt=\omega_n$.

b. Calculate the respiration rate when $x(t)=\cos(\omega_n t)$. Assume the same initial conditions as part a.